

AN ANALYTICAL METHOD FOR RESPIRATOR PERFORMANCE PREDICTION
UTILIZING THE QUANTITATIVE FIT TEST (QNFT)

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INTRODUCTION

Adequate industrial respiratory protection against toxic airborne substances is a criterion of several regulatory statutes and a hallmark of an effective industrial hygiene and respiratory protection program. The seeming endless variety of facial features, coupled with a limited repertoire of approved respirators, and the attendant fixed dimensions and/or topological contours of the facepiece, places restrictions upon the use of specific devices by an individual. The fundamental topic for assessment is that of respirator fit. A facepiece fit test is required to bound, or predict, the performance of a given respirator on a specific individual. A variety of fitting procedures have been commonly employed including visual assessment, pressure leak tests, qualitative fit test (QLFT) utilizing agents with low odor thresholds or irritating properties, and quantitative methods. The use of unambiguous instrumental techniques for measuring the fraction of a test agent which penetrates into the space created by the face and the respirator facepiece has been popularly described as a Quantitative Fit Test (QNFT).

The introduction of the Protection Factor (PF) as a numerical result of a QNFT was pioneered by Edwin C. Hyatt, et. al (1) at Los Alamos Scientific Laboratories, N.M. and has gained widespread popularity within industry and government alike. The PF is defined as the ratio of the average ambient, or challenge, agent concentration (C_a) and the breathing zone, the facepiece/face void, agent concentration (C_b).

$$PF = \frac{C_a}{C_b}$$

By popular use, the protection factor has been applied to estimate actual worker exposure given the ambient workplace concentration and to define the requisite protection, given the permissible exposure limit (PEL) of a known ambient concentration.

The PF concept, while fundamentally sound and a considerable improvement over other fit rating concepts, is accompanied by degrees of confusion and in some instances by a false sense of security. These problems usually arise as a result of equating the results of a QNFT with a PF. The results of a QNFT must be defined in such a manner as to eliminate these traditional problems.

Simply stated, a quantitative fit test is a fit leakage test, carried out in a laboratory test environment, wearing a test respirator, and is used to evaluate the performance of the test subject/respirator combination during a variety of simulated work exercises. The transfer of this data to the workplace is a function of many parameters and at best, requires the application of a safety factor to the data. If we define the results of a QNFT as a Fit Factor (FF), we can estimate the PF in terms of the FF and the safety factor (SF):

$$PF = \frac{FF}{SF}$$

The fundamental intent of this paper is to provide a logical framework in which to evaluate the magnitude of the Safety Factor. Thus, if we know, a priori, what the PF must be in order to guarantee worker safety, we can apply the SF to this number to determine the minimum acceptable FF as a result of laboratory tests, $FF = (SF)(PF)$.

The ability of a respirator to provide acceptable quality breathing air to the user is, at a bare minimum, a function of three fundamental leakage categories. The first category addresses facepiece fit and its temporal variability and use considerations. The second category relates to the efficacy of the air supply circuit which must address the composition of the hazard, the PEL and maximum use criteria assigned to the circuit. The third category addresses the respirator hardware integrity, i.e., the maintenance criteria and the presence of defective or damaged fittings, valves, facepieces, lenses, etc.

The analysis begins with a simple definition of the fit factor in terms of the important criteria of leakage rates, i.e., the volumetric breathing rate (Q_b) and the fit leakage rate (Q_f):

$$FF = \frac{Q_b}{Q_f} \quad (3)$$

By careful control over the test respirator integrity and selection of highly efficient air filtration elements, it will be shown that the Fit Factor can be measured in terms of the ambient test agent concentration and the test agent breathing zone concentration:

$$FF \text{ is approximately equal to } \frac{C_a}{C_b} \text{ test conditions} \quad (4)$$

It will be shown that for fit factors less than 1,000, the error in the approximation of equation 4 is less than 10% if a high efficiency filter of at least 99.99% efficiency is used in the QNFT.

CONCENTRATION AND LEAKAGE MEASUREMENTS

Derivation of Equivalence

A typical QNFT uses a test aerosol and the measurement instrumentation provides an output proportional to concentration. Aerosol concentrations (C) are traditionally defined in terms of the mass (m) of aerosol per unit volume (V) of air. If we differentiate such a definition with respect to time (d/dt) and assert steady state concentration conditions, then we have an alternate, rate-dependent definition in terms of the mass flowrate ($m = dm/dt$) and the volumetric air flowrate ($Q = V = dV/dt$):

$$C = \frac{m}{Q} \quad (5)$$

From a mass conservation viewpoint, the total mass flowrate in the breathing zone on inspiration (m_b) must be equal to the sum of all inlet mass flowrates (m_i) for n possible sites or paths:

$$m_b = \sum_{i=1}^n m_i \quad (6)$$

For simplicity's sake, I shall invoke isothermal, incompressible fluid conditions to eliminate the treatment of gas density changes during respiration contractions and expansions. Thus the total inspiratory breathing flowrate (Q_b) is conserved by:

$$Q_b = \sum_{i=1}^n Q_i \quad (7)$$

By reference to Appendix A, equation A8, it is shown that the mass flowrate of any path can be given in terms of the path filtration efficiency (E) and the challenge, or influent, concentration as:

$$m_i = (1 - E_i) C_i Q_i \quad (8)$$

In our present model, let us initially consider the case of three distinct categories of inlet air flow paths through the elements (e), the defects (d), and the fit leaks (f). Combining equations 5-8 with the previously defined subscripts:

$$C_b = \frac{(1 - E_e) C_e Q_e + (1 - E_d) C_d Q_d + (1 - E_f) C_f Q_f}{Q_b}$$

The influent concentration for each site may be assumed to be equal and the equivalent of the ambient test chamber concentration (C_a), such that:

$$C_e = C_d = C_f = C_a \quad (9)$$

and

$$C_b = \frac{C_a}{Q_b} ((1 - E_e) Q_e + (1 - E_d) Q_d + (1 - E_f) Q_f) \quad (10)$$

or

$$\frac{C_a}{C_b} = \frac{Q_b}{(1 - E_e) Q_e + (1 - E_d) Q_d + (1 - E_f) Q_f} \quad (11)$$

Equation 11, in this highly simplified form, demonstrates the dependence of the concentration ratio upon the various leakage site flowrates and efficiencies. The reader is well advised to note that only two of the seven variables relate to the fit leak itself.

The proper, and careful, approach to QNFT requires that we control all of the variables not relating to the fit. This is rather easily done with a few restrictions and qualifying assumptions:

- a) The test respirator, modified with a breathing zone sample port, should be carefully maintained and free from defect leakage ($Q_d=0$). If leaks are present, they will be lumped into the fit leakage ($Q_d+Q_f=Q_f$).
- b) The filtration efficiency of a hole, poor fit, or defect is assumed to be zero ($E_d=E_f=0$). This simply means that the leaks do not remove any of the test agent from the influent stream.
- c) The air purifying element, or air supply circuit, must have a very high filtration efficiency for the test agent. In the idealized case under present consideration, we let this be a perfect filter as would be experienced in a demand airline respirator ($E_e=1$ or $E_e(\%) = 100\%$).

Incorporating these conditions into equation 11 eliminates all of the annoying parameters and provides us with the expresion:

$$\frac{C_a}{C_b} = \frac{Q_b}{Q_f} \quad \text{QNFT Conditions} \quad (12)$$

In plain English, equation 12 demonstrates that the measured ratio of ambient and breathing zone concentrations in a suitably controlled test is equivalent to measuring the ratio of total inspiratory air flowrate and fit leakage flowrate, i.e., the measure of a Fit Factor as defined in equation 3. The use of equation 12 actually constitutes a relationship between the Fit Factor and the Protection Factor in a restrictive and limited sense. The fundamental importance appears as we analyze those factors which influence the Fit Factor and, as a result, directly influence the Protection Factor.

Dependence of Equivalence Upon Filtration Efficiency

The previous equivalence, equation 12, was totally dependent upon a perfect air purifying element. In the real world, we often resort to the use of non-perfect filters which introduce errors, or differences, in the concentration and flowrate measurements. If we return to equation 11 and allow some efficiency less than unity (E_e less than 1), we introduce a parallel leakage path into the respirator which bypasses the fit leak. The result is a higher breathing zone concentration than can be directly attributed to facepiece fit leakage. Thus, equation 11 is written, with the remaining QNFT provisions intact, as:

$$\frac{C_a}{C_b} = \frac{Q_b}{(1-E_e)Q_e + Q_f} \quad (13)$$

The efficiency of the element can be written in terms of a protection factor of the element (PF_e) as described in Appendix A, equation A9. Noting also that the fit leak can be written in terms of the Fit Factor (see Appendix B, equation 7), it can be shown that equation 13 reduces to:

$$\frac{C_a}{C_b} = \frac{(FF)(PF)}{FF + PF_e - 1} \quad (14)$$

A simple example of the effect or influence of a non-perfect filter on this measurement is to take a 90% efficient filter and marry it to a respirator which has a Fit Factor of 10. In this case, $PF_e=10$, $FF=10$ and $C_a/C_b=5.26$. If one were using the concentration ratio to define the Fit Factor, we would incorrectly rate the fit by a factor of nearly one-half. The manner in which this error is introduced is a conservative one in treating the fit leakage, i.e., we say that the fit leak is greater than it actually is.

The Error in Fit Leakage Measurement Using High Efficiency Particulate Filters

A typical solution to the QNFT filtration efficiency problem is to employ high efficiency particulate filters which meet or exceed minimum NIOSH Certification Criteria (2) for the DOP Penetration Tests. This criterion establishes a minimum efficiency of 99.97% when tested with a monodispersed 0.3 um diameter DOP aerosol. This is equivalent to a maximum penetration of 0.03% and often results in penetrations less than 0.01%. The corresponding element protection factors are 3333 and greater than 10,000 respectively. As previously shown, equation 12, the fit test actually measures a concentration ratio which we shall define as a hybrid fit factor (FF^*):

$$FF^* = \frac{C_a}{C_b} \quad \text{QNFT} \quad (15)$$

The fractional difference (Delta), or deviation, from the true Fit Factor may be computed by:

$$\Delta = \frac{FF^* - FF}{FF^*} \quad (16)$$

$$\Delta = \frac{1 - FF}{PF_e} \quad (17)$$

Appendix B provides a complete derivation of equation 17 and tabulates Delta for filter penetrations between 1% and 0.01%. Figure 1 is an abstraction of this data for two filter penetrations and Fit Factors up to 1,000. Several observations can be readily made concerning the deviation, or error:

- a) The use of non-perfect filters consistently over estimates the fit leakage and is thereby a conservative approach to QNFT.
- b) Use of minimal NIOSH approved high efficiency filters ($E_e=99.97\%$) introduces no more than a 30% over estimate of fit leakage for true Fit Factors less than or equal to 1,000.
- c) The use of a 99.99% efficient filter will assure an error no larger than 10% in assessment of true Fit Factors throughout the range from unity to 1,000 as measured by a concentration ratio (FF^*).

QNFT WITHOUT HIGH EFFICIENCY FILTERS

As is often the case, one may wish to fit test using the QNFT on a respirator which may not have a high particulate filtration efficiency for the test aerosol. Good examples of this situation are the quarter mask and disposable dust respirators, gas and vapor respirators, and facepieces with replaceable elements which are not available in high efficiency models. All is not lost for this category of users. While it may be true that they may not be able to carry out a QNFT with the same degree of accuracy as described in the preceding section, it is possible to bound the problem. If we remember that non-perfect filters always over-estimate fit leakage in a QNFT, then the measurement of FF^* is the absolute worst possible case. Let us, however, go one further step. If one actually measures the test aerosol penetration through the element over an appropriate range of flowrates in a suitable test apparatus, we will have a number for PF_e using the aerosol. Re-arranging equation 14 provides us with a method for computing the Fit Factor as a function of FF^* and PF_e :

$$FF = \frac{FF^* (PF_e - 1)}{PF_e - FF^*} \quad (18)$$

A specific example may serve to illustrate this approach.

FIGURE 1

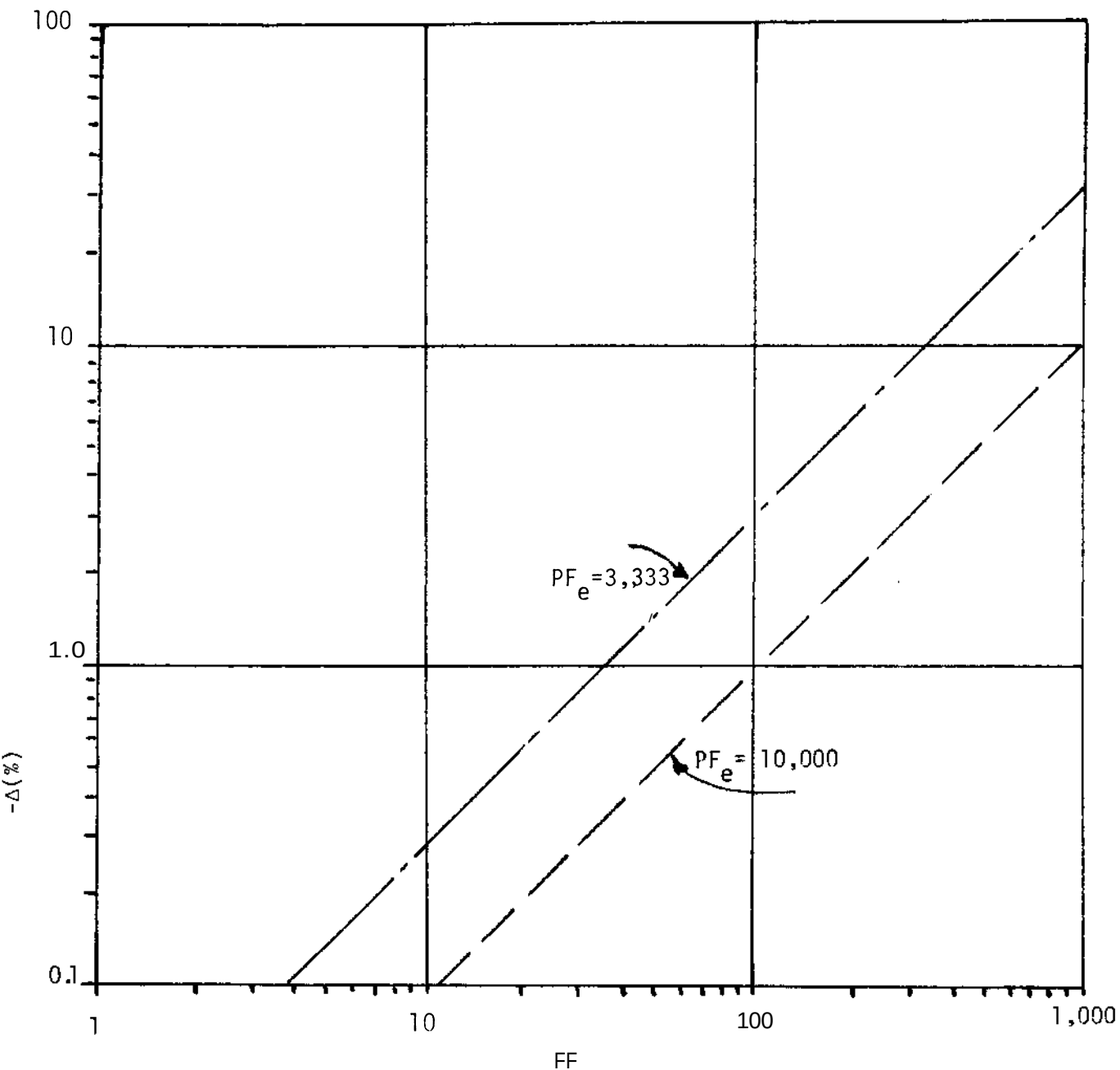


FIGURE 1

The Error $\Delta(\%)$ in Fit Factor (FF) Measurement Using Concentration Ratios for Two Different Filter Protection Factors (PF_e).

Assume that we measure the aerosol penetration on a disposable dust filter media for flowrates and find that it varies between 10% and 15%. The best PF calculated would be 10. We now carry out a QNFT and determine that the average penetration was 15% corresponding to $FF^*=6.667$. One could argue that all of the penetration was due to the element; however, if the difference between 15% and 10% was due to fit leakage, then equation 18 predicts that the true Fit Factor must be

$$FF = \frac{FF^*(PF_e - 1)}{PF_e - FF^*} = \frac{6.667(10 - 1)}{10 - 6.667} = 18 \quad (19)$$

The usual approach to this problem is to add additional filtration media to the test respirator to maximize PF_e and thus improve resolution of the Fit Factor measurement for the required range of interest of fit.

Fit Factor Dependence Upon Breathing Resistance

The flowrate through a resistance can be analytically represented as a function of the pressure drop (Delta P) by:

$$Q_i = k_i \Delta P^{a_i} \quad (20)$$

The flow coefficient (k) represents dimension of the leak and various unit conversion factors while the exponent (a) characterizes the type of flow. Pipes, capillaries, and filters generally favor the first power ($a=1$) while pinholes, orifices and other abrupt, short length obstructions approach the one-half power ($a=0.5$).

Returning to the definition of Fit Factor (eq.3) and eliminating defects, the inspiratory air flowrate is the sum of the element and fit leak flowrates.

$$FF = \frac{Q_b}{Q_f} = \frac{Q_e + Q_f}{Q_f} = \frac{Q_e}{Q_f} + 1 \quad (21)$$

and by replacement with equation 19 and for FF greater than 1. we have:

$$FF \text{ is approximately equal to } \frac{k_e \Delta P^{(a_e - a_f)}}{k_f} \quad (22)$$

In this form, we see that FF is strongly dependent upon the flow coefficients and the Delta P across the respirator. A unique solution occurs when the power law dependence of the filter is equal to that of the fit leak, $a_e = a_f$ or $a_e - a_f = 0$. Under these simplified conditions,

$$FF = \frac{k_e}{k_f} \quad (23)$$

and the Fit Factor is independent of the flowrate. The reader will observe that most fit test scenarios involve unequal exponents, $a_e \neq a_f$, which result in a pressure drop, flowrate dependent penetrations characterized by

fluctuating breathing zone concentrations. Elastic deformation and movement of both face and facepiece results in dynamic responses to respiration which change not only the flow coefficient, but the exponent as well. Notwithstanding these complications from an analytical model standpoint, one can derive useful information from the simple form of equation 22. For this unique condition we note that a QNFT determines an average value of k_f by carrying out a variety of motions and breathing exercise on a respirator whose filter element is characterized by a constant k_e . If we now change the element flow coefficient by exchanging filters, leaving the fit intact, we would not expect the average property of the leak to change. We would expect the element to vary by as much, or as many, as we have differing flow coefficient elements. We likewise would expect a commensurate variability in elements.

Table 1 provides selected flow coefficients for negative pressure respirators which have been NIOSH approved (2). The coefficient is calculated based upon the form of equation 22 and a first power law dependence. Large flow coefficients correspond to low breathing resistance and vice versa. Initial and final breathing resistance corresponds to conditions before and after acceptable loading or like tests. If we take the ratio of the highest to the lowest tabulated flow coefficients, 7.083 and 1.063, we have the range of variability in Fit Factor for a given, constant facepiece fit leak. This ratio is a little less than 7 in this, presumably, worst case. If we look at the last column of Table 1, k_0/k_1 , we are able to see that the expected change in Fit Factor would be throughout the usable life of the element. The worst case here is a little less than 3 for the re-useable dust and mist and fume filter.

We can see that it very well pays to take breathing resistance into account when carrying out a QNFT. The same holds true for the condition of filters in use in the workplace. At the very least, we must take these uncertainties into account when we estimate a protection factor for the user.

TABLE 1

SELECTED INITIAL AND FINAL BREATHING RESISTANCE FOR 30 CFR 11
APPROVED NEGATIVE PRESSURE RESPIRATORS AT 85 LITER PER MINUTE AIR FLOWRATE

Type	Initial Resistance		Final Resistance		Ratio
	Delta P ₀	k_0	Delta P ₁	k_1	
				k_0/k_1	
Chin-Type Gas Mas (PF)	65	1.308	80	1.063	1.23
Single Use DMF Resp.	12	7.083	15	5.667	1.25
Single Use DMF Filter	30	2.833	50	1.700	1.67
Reuseble DMF Filter	20	4.250	40	2.125	2.0
Gases/Vapors (NF)	40	2.125	45	1.889	1.125
Gases/Vapors (PF)	50	1.700	70	1.214	1.40

Notes: * PF - with particulate filter

 NF - no particulate filter

 DMF - dusts, mists, and fume

 Delta P - pressure drop in mm. of water column

 k - flow coefficient--see equation 19

 o - subscript applies to new filter

 1 - subscript applies to filter after loading test

Tests are applicable to respirator assembly with filters installed. If the tests are run on the filter separately, one can infer that the pressure drop applies to the filter as long as the facepiece does not have additional flow restrictions in the air path. Thus, if elements are used in pairs, the pressure drop should be equal or matched and the flow through each filter is 42.5 l/min.

VARIABILITY OF FIT

A significant number of users may observe a rather large inter-day variability of the measured Fit Factor. The minimization of this variance is assisted by observing breathing resistance constraints, proper mask maintenance, and filtration element efficiency as previously discussed. From this point, the focus must be upon proper selection and adequate training. Many times one introduces extreme variability by not properly exercising the fit during the QNFT. An example of this condition is the case of a poor to fair fitting respirator which can be sealed to the face for a static test. If the test subject's movements are slow and easy, it might be possible to carry out the QNFT and achieve a high Fit Factor, say 8,000. Under typical use and donning practices, the respirator may only provide Fit Factors of 50. The one atypical test which yielded the Fit Factor of 8,000 introduced a variability defined by the ratio of 8,000/50 or 160.

Leidel (3) has discussed the variance problem in depth and the exact nature of the variability distribution is still a problem for debate and further research. The American National Standard Practices for Respiratory Protection ANSI Z88.2 (1980) has considered the problem as demonstrated by the recommendation that the lowest of three independently measured Protection (or Fit) Factors be used as a criterion of a QNFT series. From a practical, user oriented standpoint there is not enough time available to accurately determine the Fit Factor distribution for those who are required by regulatory statute to implement a QNFT program. The statistical sample of daily fit would be inappropriately large.

Employing only the simplest concepts of probability and statistics and preserving the ANSI three-test approach, it is evident that some conservation is justified. Consider that whatever the shape of the distribution function is, it must have a median value. One single QNFT has a 50% chance of measuring a FF which is above the median. Likewise, we have a 25% and 12.5% chance of having two and three consecutive QNFT's falling above the median respectively. The remainder of the chances must include those tests which have at least one value falling below the median. This remainder of 87.5% for a three-test series is reasonable but it does imply that one out of every eight persons tested will be assigned a Fit Factor which is above the median value, thus consistently over-estimating the protection available. Any attempt to establish a meaningful measure of conservatism will most certainly invoke a heated debate, and, in the absence of extensive data, is beyond the scope of this paper.

I will, however, propose (briefly) a number which has some basis in experimental data which can be easily be changed as more knowledge is gained. Unfortunately, to propose a number, I fall into the trap of being forced to assume that Fit Factors are distributed in a pseudo log-normal fashion. The cumulative log normal distribution function, plotted on log probability paper, is a straight line. The slope of this line is characterized by the geometric standard deviation (σ_g) which is defined as the ratio of the Fit Factor at one standard deviation (σ) above the median ($FF(\sigma)$) to the median value (FF). The average σ_g for Hyatt's data as summarized by Leidel (Tables 14 and 15 or reference 3) tends to increase from about 5 for a three-test series to approximately 8 for a six-test series. For higher numbers of tests, the value does not appreciably increase; however, the data is too limited to rely upon heavily. The fact that the σ_g does increase means that three tests are inadequate to define the distribution functions.

John Pritchard (4) has reported on a geometric standard deviation closer to 5 for mixed n-test trials. Neither of the reported data sets have recognized all the necessary controls to test for fit variability only. Thus, one might propose, in the interim, that fit variability is characterized by a geometric standard deviation less than or equal to 5.

Returning to the implication that 12.5% of the people completing a three-test series have a very high probability that all three test scores fall above the median, I propose that this is an unacceptable risk. In lieu of any definitive criteria, I propose to increase the odds of at least one test falling below the median from 87.5% to 99%. Resorting to the standard statistical tables for normal distributions and assuming a log-normal distribution, we find that 99% of the time, the lowest of three successive trials will fall above the median times $0.785 \sigma_g$. This product is approximately equal to 4. In other words, 99% of the time the lowest of three measured fit factors will be less than or equal to 4 times the median value. Thus, if we divide the lowest factor by 4, we conclude that this new factor is at least as low as the median value. Scaling by another half standard deviation, i.e., $.5 \sigma_g = 2.5$, we assure assignment of a lowest fit factor which is well below the median for a continuous distribution. The cumulative affect of these two "corrections" to the lowest of three test scores is to divide by 10.

CONCLUSIONS AND REMARKS

It has been demonstrated that a Quantitative Fit Test measures a Fit Factor which is representative of the ratio of breathing to fit leakage flowrates. The QNFT must take into account the test filter efficiency if aerosol concentration measurements are to be valid representations of fit leakage. The breathing resistance also introduces a factor for consideration in that it directly affects the measurement of a Fit Factor. The lowest of three Fit Factors provides no guarantee of conservatism and must be considered in the variability of fit assessment. A safety factor can be assembled which takes into account:

- a. Low efficiency filtration elements
- b. Anticipated breathing resistance changes
- c. Variability of fit

This safety factor can then be applied to the Fit Factor to give a reasonable estimate of the workplace Protection Factor. A typical safety factor could be on the order of 20, determined by a product of 2 for resistance and 10 for variability of fit.

If we consider that the maximum recommended Protection Factor (ANSI Z88.2 (1980) for a half mask is 100, the safety factor would suggest that the lowest of three Fit Factors should be in the range of 2000. While this number may appear large, it is not impractical. Bearing in mind the fact that only the best fitting respirators should be worn for the highest allowable protection, we ask if 2000 is unreasonable. The answer is no! Half mask respirators demonstrate Fit Factors well above this value in a large percentage of cases. The problem becomes simply one of finding the respirator for a given individual which meets this criterion. The purpose, I might add, of Quantitative Fit Testing in the first place.

REFERENCES

1. Edwin C. Hyatt et.al., "Respirator Protection Factors" Los Alamos Scientific Laboratory Report LA-6084-MS, VC-41, January, 1976.
2. Title 30, Code of Federal Regulations Part 11, Respiratory Protective Apparatus; Tests for Permissibility, Fees.
3. Leidel, Nelson A. and William A. Burgess, "Performance of Facesal Fit Tests for Respiratory Protection", presented to the American Industrial Hygiene Conference, Houston, Texas, May 20, 1980.
4. John Pritchard, Health Resources Management Limited, Santa Fe, New Mexico, Private Communication.

Appendix A

Circuit Mass Flowrate of Agent

A filtration system or element reduces the concentration of airborne agent passing through the element. The defining conservation equation relating concentration (C), mass flowrate ($\dot{m}$) and volumetric gas flowrate (Q) is given by:

$$\dot{m} = CQ \quad (1)$$

The inlet or influent mass flowrate ($\dot{m}_i$) is either retained by the filter ($\dot{m}_r$) or carried through to the effluent or output side ($\dot{m}_o$). The conservation relationship is simply:

$$\dot{m}_i = \dot{m}_r + \dot{m}_o \quad (2)$$

The normalized form of equation 2 defines the fraction retained (efficiency) and the fraction penetrating;

$$\frac{\dot{m}_r}{\dot{m}_i} = \frac{\dot{m}_r}{\dot{m}_i} + \frac{\dot{m}_o}{\dot{m}_i} \quad (3)$$

$$1 = E + P \quad (4)$$

where efficiency (E) and penetration (P) are defined respectively in the two right hand terms of equation 3.

The determination of filter performance can be made by either efficiency or penetration measurements. Direct efficiency measurements require gravimetric (weighing) techniques to actually measure the retained mass of agent on the filter, and in general must be carried out over long time intervals to allow the retained fraction to accumulate to some weighable and significant percentage of the filter tare weight. This procedure also involves the collection of all of the output mass fraction to satisfy solution of equation 3. An alternate method of rating filter performance is the penetration measurement which can be carried out by a real time concentration measurement set. We note that the penetrating fraction can be expanded using equation 1 to provide:

$$P = \frac{\dot{m}_o}{\dot{m}_i} = \frac{C_o}{C_i} \frac{Q}{Q} = \frac{C_o}{C_i} \quad (5)$$

and $C_o = C_i P = C_i (1-E) \quad (6)$

Appendix B Fit Leak and Factor Relationships

From the definition of Fit Factor (FF) as a simple ratio of total inspiratory breathing flowrate (Q_b) and fit leak rate (Q_f), we can write the fit leak as:

$$FF = \frac{Q_b}{Q_f} \quad (1)$$

$$Q_f = \frac{Q_b}{FF} \quad (2)$$

The expression for ambient to breathing concentrations

$$\frac{C_a}{C_b} = \frac{Q_b}{(1-E_e)Q_e + Q_f}$$

can be rewritten in terms of equation 2 and the relationship,

$$PF_e = \frac{1}{(1-E_e)} \quad (3)$$

as a new expression which removes the flowrate terms and simplifies as follows; noting that $Q_b = Q + Q_f$:

$$\frac{C_a}{C_b} = \frac{Q_b}{(1-E_e)(Q_b - Q_f) + Q_f} \quad (4)$$

$$\frac{C_a}{C_b} = \frac{Q_b}{\frac{(Q_b - Q_b/FF) + Q_b/FF}{PF_e}} \quad (5)$$

$$\frac{C_a}{C_b} = \frac{(FF)(PF_e)Q_b}{(FF)Q_b - Q_b + PF_e Q_b} \quad (6)$$

$$\frac{C_a}{C_b} = \frac{(FF)(PF_e)}{FF + PF_e - 1} \quad (7)$$

If we allow the concentration ratio in equation 7 to be a hybrid fit factor (FF^*), we can calculate the differential error (A) from:

$$\Delta = \frac{FF^* - FF}{FF} \quad (8)$$

$$FF^* = \frac{(FF)(PF_e)}{FF + PF_e - 1} \quad (9)$$